

Comments on CP, T and CPT Violation in Neutral Kaon Decays

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Abstract

We comment on CP, T and CPT violation in the light of interesting new data from the CPLEAR and KTeV Collaborations on neutral kaon decay asymmetries. Other recent data from the CPLEAR experiment, constraining possible violations of CPT and the $\Delta S = \Delta Q$ rule, exclude the possibility that the semileptonic-decay asymmetry A_T measured by CPLEAR could be solely due to CPT violation, confirming that their data constitute direct evidence for T violation. The CP-violating asymmetry in $K_L \rightarrow e^- e^+ \pi^- \pi^+$ recently measured by the KTeV Collaboration does not by itself provide direct evidence for T violation, but we use it to place new bounds on CPT violation.

1 Introduction

Ever since the discovery of CP violation in $K_L^0 \rightarrow 2\pi$ decay by Christenson, Cronin, Fitch and Turlay in 1964 [1], its understanding has been a high experimental and theoretical priority. Until recently, mixing in the $K^0 - \bar{K}^0$ mass matrix was the only known source of CP violation, since it was sufficient by itself to explain the observations of CP violation in other $K_{S,L}^0$ decays, no CP violation was seen in experiments on K^\pm , charm or B -meson decays, and searches for electric dipole

moments only gave upper limits [2]. There has in parallel been active discussion whether the observed CP violation should be associated with the violation of T or CPT [3]. Stringent upper limits on CPT violation [4] in the $K^0 - \bar{K}^0$ system have been given [5], in accord with the common theoretical prejudice based on a fundamental theorem in quantum field theory [6]. This suggests strongly that T must be violated, but, at least until recently, there was no direct observation of T violation. An indirect demonstration of T violation in neutral kaons, based on a phenomenological analysis of CP-violating amplitudes, was made in 1970 using data on the decay of long- and short-lived kaons into two neutral pions [7]. However, that analysis assumed unitarity, namely that kaons disappeared only into the observed states.

The accumulation of experimental observations of CP and T violation has accelerated abruptly in the past few months. There have been two results on K^0, \bar{K}^0 decays for which interpretations as direct observations of T violation have been proposed. One is an asymmetry in $p\bar{p}$ annihilation, $p\bar{p} \rightarrow K^- \pi^+ K^0$ or $K^+ \pi^- \bar{K}^0$ [8], and the other is a T-odd angular asymmetry in $K_L^0 \rightarrow \pi^+ \pi^- e^+ e^-$ decay [9]. More recently, a tantalizing hint has been presented that CP may be violated at a high level in $B^0 \rightarrow J/\psi K_S$ decays [10]. Most recent of all, a previous measurement of direct CP violation in the amplitudes for $K_{S,L}^0 \rightarrow 2\pi$ decays [11] has now been confirmed by the KTeV Collaboration [9], providing an improved determination of a second independent CP-violating experimental number, namely ϵ'/ϵ , to test theories and to discriminate between them.

One casualty of this measurement of ϵ'/ϵ has been the superweak theory [12], according to which all CP violation should be ascribed to mass mixing in the $K^0 - \bar{K}^0$ system. Still surviving is the Kobayashi-Maskawa model of weak charged-current mixing within the Standard Model with six quarks [13]. Indeed, the new KTeV result arrives 23 years after ϵ'/ϵ was first calculated within the Kobayashi-Maskawa model [14], and it was pointed out that this would be a (difficult) way to discriminate between this and the superweak theory, providing (at least part of) the motivation for this experiment. Coincidentally, the value estimated there agrees perfectly with the current world average for ϵ'/ϵ , although many new diagrams and numerical improvements have intervened [15]. The latest theoretical wisdom about the possible value of ϵ'/ϵ within the Standard Model is consistent with the value measured, at least if the strange-quark mass is sufficiently small [16]. Thus ϵ'/ϵ does not cry out for any extension of the Standard Model, such as supersymmetry [17], though this cannot be excluded.

It is not the purpose of this article to review in any detail the potential significance of the ϵ'/ϵ measurements, or of the hint of a CP-violating asymmetry in $B^0 \rightarrow J/\psi K_S$. Rather, we wish to comment on the suggested interpretations of the asymmetry in $p\bar{p} \rightarrow K^- \pi^+ K^0$ and $K^+ \pi^- \bar{K}^0$ [8], and of the T-odd angular asymmetry

in $K_L^0 \rightarrow \pi^+\pi^-\ell^+\ell^-$ as possible direct evidence for T violation [9]. We argue that the former can indeed be interpreted in this way, when combined with other CPLEAR data constraining the possible violation of the $\Delta S = \Delta Q$ rule and CPT violation in semileptonic K^0 decays [18,19]. We use the $K_L^0 \rightarrow \pi^+\pi^-\ell^+\ell^-$ decay asymmetry as a novel test of CPT invariance in decay amplitudes, though one that may not yet be comparable in power with other tests of CPT.

The layout of this article is as follows: in Section 2 we first introduce the semileptonic-decay asymmetry recently measured by CPLEAR, then in Section 3 we introduce a density-matrix description that includes a treatment of unstable particles as well as allowing for the possibility of stochastic CPT violation [20–22]. In Section 4 we apply this framework to show that the CPLEAR asymmetry cannot be due to CPT violation, and is indeed a direct observation of T violation. We also comment whether other examples of CP violation can be mimicked by CPT violation [22]. Then, in Section 5 we analyze the decay asymmetry observed by the KTeV collaboration, arguing that it does not have an unambiguous interpretation as a direct observation of T violation. It could not be due to CPT violation in the mass-mixing matrix, but could in principle be due to ‘direct’ CPT violation in a decay amplitude.

2 The CPLEAR Asymmetry in $p\bar{p} \rightarrow K^-\pi^+K^0$ and $K^+\pi^-\bar{K}^0$

We first recall briefly the key features of the asymmetry A_T observed by CPLEAR, motivating its interpretation as direct evidence for T violation. The essential idea is to look for a violation of reciprocity in the rates for $K^0 \rightarrow \bar{K}^0$ and the time-reversed reaction $\bar{K}^0 \rightarrow K^0$, denoted by $P_{K\bar{K}}$ and $P_{\bar{K}K}$, respectively, as expressed in the asymmetry

$$A_T \equiv -\frac{P_{K\bar{K}} - P_{\bar{K}K}}{P_{K\bar{K}} + P_{\bar{K}K}} \quad (1)$$

CPLEAR has the unique capability to tag the initial K^0 or \bar{K}^0 by observing an accompanying $K^\pm\pi^\mp$ pair in a $p\bar{p}$ annihilation event. However, it is also necessary to tag the K^0 or \bar{K}^0 at some later time, which CPLEAR accomplishes using semileptonic decays, and constructing the observable asymmetry [8]:

$$A_l \equiv -\frac{R[\pi^+K^-\pi^+e^-\bar{\nu}] - R[\pi^-K^+\pi^-e^+\nu]}{R[\pi^+K^-\pi^+e^-\bar{\nu}] + R[\pi^-K^+\pi^-e^+\nu]} \quad (2)$$

where rates are denoted by R . If one assumes the $\Delta S = \Delta Q$ rule, whose validity has been confirmed independently by CPLEAR (see below), then (2) may be re-expressed

as

$$A_l = \frac{P_{\overline{K}K}(\tau)BR[K^0 \rightarrow \pi^- e^+ \nu] - P_{K\overline{K}}(\tau)BR[\overline{K}^0 \rightarrow \pi^+ e^- \overline{\nu}]}{P_{\overline{K}K}(\tau)BR[K^0 \rightarrow \pi^- e^+ \nu] + P_{K\overline{K}}(\tau)BR[\overline{K}^0 \rightarrow \pi^+ e^- \overline{\nu}]} \quad (3)$$

where decay branching ratios are denoted by BR . In our discussion below, we consider both the cases where the $\Delta S = \Delta Q$ rule is assumed and where it is relaxed ¹.

If one assumes CPT invariance in the semileptonic-decay amplitudes, as was done in the CPLEAR analysis [8], then $A_l = A_T$ and the asymmetry observed by CPLEAR can be interpreted as T violation. Some doubts about this interpretation have been expressed [24], apparently based on concerns about the inapplicability of the reciprocity arguments of [25] to unstable particles. We do not believe this to be a problem, since the analysis of [25] can be extended consistently to include unstable particles [22,23,26].

However, it has also been proposed [26] that one might be able to maintain T invariance, $P_{\overline{K}K} = P_{K\overline{K}}$, interpreting the asymmetry observed by CPLEAR instead as CPT violation in the semileptonic-decay amplitudes [26]. This interpretation of the CPLEAR result would be more exciting than the conventional one in terms of T violation. It was suggested in [26] that this hypothesis of CPT violation could be tested in the semileptonic decays $K_S \rightarrow \pi l \nu$. However, the hypothesis of [26] can, in fact, already be excluded by other published CPLEAR data, as we see below.

3 Density-Matrix Formalism

Before discussing this in more detail, we review the density-matrix formalism [22], which is a convenient formalism for treating unstable particles, and enables us to present a unified phenomenological analysis including also the possibility of stochastic CPT violation associated with a hypothetical open quantum-mechanical formalism associated with some approaches to quantum gravity [27,20,28,21]. In fact, as we recall below, this formalism has already been used in the Appendix of [22] to discard the possibility that CP violation in the neutral-kaon system could be ‘mimicked’ by the CPT-violating mass-matrix parameter δ within conventional quantum mechanics. As we discuss later, this was possible only if

$$\text{Re}(\delta) \sim (1.75 \pm 0.7) \times 10^{-3} \quad (4)$$

¹ A recent theoretical discussion using the $\Delta S = \Delta Q$ rule and CPT invariance is given in [23].

This analysis is also reviewed briefly below, taking into account recent data of the CPLEAR collaboration [18] on $\text{Re}(\delta)$, which were not available at the time of writing of [22], and exclude the possibility (4).

When one considers an unstable-particle system in isolation, without including its decay channels, its time-evolution is non-unitary, so one uses a non-Hermitean effective Hamiltonian: $H \neq H^\dagger$. The temporal evolution of the density matrix, ρ , is given within the conventional quantum-mechanical framework by:

$$\partial_t \rho = -i(H\rho - \rho H^\dagger) . \quad (5)$$

In the case of the neutral-kaon system, the phenomenological Hamiltonian contains the following Hermitean (mass) and anti-Hermitean (decay) components:

$$H = \begin{pmatrix} (M + \frac{1}{2}\delta M) - \frac{1}{2}i(\Gamma + \frac{1}{2}\delta\Gamma) & M_{12}^* - \frac{1}{2}i\Gamma_{12}^* \\ M_{12} - \frac{1}{2}i\Gamma_{12} & (M - \frac{1}{2}\delta M) - \frac{1}{2}i(\Gamma - \frac{1}{2}\delta\Gamma) \end{pmatrix} , \quad (6)$$

in the (K^0, \bar{K}^0) basis. The δM and $\delta\Gamma$ terms violate CPT. Following [20], we define components of ρ and H by

$$\rho \equiv \frac{1}{2}\rho_\alpha \sigma_\alpha \quad ; \quad H \equiv \frac{1}{2}h_\alpha \sigma_\alpha \quad ; \quad \alpha = 0, 1, 2, 3 \quad (7)$$

in a Pauli σ -matrix representation : since the density matrix must be Hermitean, the ρ_α are real, but the h_β are complex in general.

We may represent conventional quantum-mechanical evolution by $\partial_t \rho_\alpha = H_{\alpha\beta} \rho_\beta$, where, in the (K^0, \bar{K}^0) basis and allowing for the possibility of CPT violation,

$$H_{\alpha\beta} \equiv \begin{pmatrix} \text{Im}h_0 & \text{Im}h_1 & \text{Im}h_2 & \text{Im}h_3 \\ \text{Im}h_1 & \text{Im}h_0 & -\text{Re}h_3 & \text{Re}h_2 \\ \text{Im}h_2 & \text{Re}h_3 & \text{Im}h_0 & -\text{Re}h_1 \\ \text{Im}h_3 & -\text{Re}h_2 & \text{Re}h_1 & \text{Im}h_0 \end{pmatrix} . \quad (8)$$

It is convenient for the rest of our discussion to transform to the $K_{1,2} = \frac{1}{\sqrt{2}}(K^0 \mp \bar{K}^0)$ basis, corresponding to $\sigma_1 \leftrightarrow \sigma_3$, $\sigma_2 \leftrightarrow -\sigma_2$, in which $H_{\alpha\beta}$ becomes

$$H_{\alpha\beta} = \begin{pmatrix} -\Gamma & -\frac{1}{2}\delta\Gamma & -\text{Im}\Gamma_{12} & -\text{Re}\Gamma_{12} \\ -\frac{1}{2}\delta\Gamma & -\Gamma & -2\text{Re}M_{12} & -2\text{Im}M_{12} \\ -\text{Im}\Gamma_{12} & 2\text{Re}M_{12} & -\Gamma & -\delta M \\ -\text{Re}\Gamma_{12} & -2\text{Im}M_{12} & \delta M & -\Gamma \end{pmatrix}. \quad (9)$$

The corresponding equations of motion for the components of ρ in the $K_{1,2}$ basis are given in [22].

The CP-violating mass-mixing parameter ϵ and the CPT-violating mass-mixing parameter δ are given by ².

$$\epsilon = \frac{\text{Im}M_{12}}{\frac{1}{2}|\Delta\Gamma| + i\Delta m} = |\epsilon|e^{-i\phi}, \quad \delta = -\frac{\frac{1}{2}\delta\Gamma + i\delta M}{\frac{1}{2}|\Delta\Gamma| + i\Delta m}. \quad (10)$$

One can readily verify [22] that ρ decays at large t to

$$\rho \sim e^{-\Gamma_L t} \begin{pmatrix} 1 & \epsilon^* + \delta^* \\ \epsilon + \delta & |\epsilon + \delta|^2 \end{pmatrix}, \quad (11)$$

which has a vanishing determinant, thus corresponding to a pure long-lived mass eigenstate K_L , whose state vector is

$$|K_L\rangle \propto (1 + \epsilon - \delta) |K^0\rangle - (1 - \epsilon + \delta) |\bar{K}^0\rangle \quad (12)$$

Conversely, in the short-time limit a K_S state is represented by

$$\rho \sim e^{-\Gamma_S t} \begin{pmatrix} |\epsilon - \delta|^2 & \epsilon - \delta \\ \epsilon^* - \delta^* & 1 \end{pmatrix}, \quad (13)$$

² We follow here the conventions of [22], which are related to the notation used elsewhere [29] for CP- and CPT-violating parameters by $\epsilon = -\epsilon_M^*$, $\delta = -\Delta^*$, with $*$ denoting complex conjugation. Thus the superweak angle ϕ_{sw} defined in [29] is related to the angle ϕ in (10) by $\phi = \phi_{sw} - \pi$, so that $\tan\phi_{sw} = \tan\phi = 2\Delta m/|\Delta\Gamma|$.

which also has zero determinant and hence represents a pure state:

$$|K_S\rangle \propto (1 + \epsilon + \delta) |K^0\rangle + (1 - \epsilon - \delta) |\bar{K}^0\rangle \quad (14)$$

Note that the relative signs of the δ terms have reversed between (11) and (13): this is the signature of mass-matrix CPT violation in the conventional quantum-mechanical formalism, as seen in the state vectors (12) and (14).

The differential equations for the components of ρ may be solved in perturbation theory in $|\epsilon|$ and the new parameters

$$\widehat{\delta M} \equiv \frac{\delta M}{|\Delta\Gamma|}, \quad \widehat{\delta\Gamma} \equiv \frac{\delta\Gamma}{|\Delta\Gamma|}. \quad (15)$$

To first order, one finds [22]:

$$\begin{aligned} \rho_{11}^{(1)} &= -2|X'| |\rho_{12}(0)| \left[e^{-\Gamma_L t} \cos(\phi - \phi_{X'} - \phi_{12}) - e^{-\Gamma t} \cos(\Delta m t + \phi - \phi_{X'} - \phi_{12}) \right] \\ \rho_{22}^{(1)} &= -2|X| |\rho_{12}(0)| \left[e^{-\Gamma_S t} \cos(\phi + \phi_X + \phi_{12}) - e^{-\Gamma t} \cos(\Delta m t - \phi - \phi_X - \phi_{12}) \right] \\ \rho_{12}^{(1)} &= \rho_{11}(0) |X| e^{-i(\phi + \phi_X)} \left[e^{-\Gamma_L t} - e^{-(\Gamma + i\Delta m)t} \right] \\ &\quad + \rho_{22}(0) |X'| e^{i(\phi - \phi_{X'})} \left[e^{-\Gamma_S t} - e^{-(\Gamma + i\Delta m)t} \right] \end{aligned} \quad (16)$$

where the two complex constants X and X' are defined by:

$$\begin{aligned} X &= |\epsilon| + \frac{1}{2} \cos \phi \widehat{\delta\Gamma} + i \cos \phi \widehat{\delta M}, \quad \tan \phi_X = \frac{\cos \phi \widehat{\delta M}}{|\epsilon| + \frac{1}{2} \cos \phi \widehat{\delta\Gamma}}, \\ X' &= |\epsilon| - \frac{1}{2} \cos \phi \widehat{\delta\Gamma} + i \cos \phi \widehat{\delta M}, \quad \tan \phi_{X'} = \frac{\cos \phi \widehat{\delta M}}{|\epsilon| - \frac{1}{2} \cos \phi \widehat{\delta\Gamma}}. \end{aligned} \quad (17)$$

The special case that occurs when $\delta M = 0$ and $|\epsilon| = 0$, namely

$$\begin{aligned} \delta\Gamma > 0 : \phi_X &= 0, \quad \phi_{X'} = \pi \\ \delta\Gamma < 0 : \phi_X &= \pi, \quad \phi_{X'} = 0. \end{aligned} \quad (18)$$

will be of particular interest for our purposes.

With the results for ρ through first order, and inserting the appropriate initial conditions [22] we can immediately write down expressions for various observables [22]

of relevance to CPLEAR. The values of observables O_i are given in this density-matrix formalism by expressions of the form [20]

$$\langle O_i \rangle \equiv \text{Tr} [O_i \rho] , \quad (19)$$

where the observables O_i are represented by 2×2 Hermitean matrices. Those associated with the decays of neutral kaons to 2π , 3π and $\pi l \nu$ final states are of particular interest to us. If one assumes the $\Delta S = \Delta Q$ rule, their expressions in the $K_{1,2}$ basis are

$$\begin{aligned} O_{2\pi} &\propto \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} , & O_{3\pi} &\propto \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} , \\ O_{\pi-l+\nu} &\propto \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} , & O_{\pi+l-\bar{\nu}} &\propto \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} . \end{aligned} \quad (20)$$

which constitute a complete Hermitean set. We consider later the possible relaxation of the $\Delta S = \Delta Q$ rule, and also the possibility of direct CPT violation in the observables (20), which would give them different normalizations. The small experimental value of ϵ'/ϵ would be taken into account by different magnitudes for $O_{2\pi}$ in the charged and neutral modes, but we can neglect this refinement for our purposes.

In this formalism, pure K^0 or \bar{K}^0 states, such those provided as initial conditions in the CPLEAR experiment, are described by the following density matrices:

$$\rho_{K^0} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} , \quad \rho_{\bar{K}^0} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} . \quad (21)$$

We note the similarity of the above density matrices (21) to the representations (20) of the semileptonic decay observables, which reflects the strange-quark contents of the neutral kaons and our assumption of the validity of the $\Delta S = \Delta Q$ rule: $K^0 \ni \bar{s} \rightarrow \bar{u} l^+ \nu$, $\bar{K}^0 \ni s \rightarrow u l^- \bar{\nu}$.

4 Interpretation of the CPLEAR Asymmetry

In the CPLEAR experiment [8], the generic quantities measured are asymmetries of decays from an initially pure K^0 beam as compared to the corresponding

decays from an initially pure \bar{K}^0 beam:

$$A(t) = \frac{R(\bar{K}_{t=0}^0 \rightarrow \bar{f}) - R(K_{t=0}^0 \rightarrow f)}{R(\bar{K}_{t=0}^0 \rightarrow \bar{f}) + R(K_{t=0}^0 \rightarrow f)}, \quad (22)$$

where $R(K_{t=0}^0 \rightarrow f) \equiv \text{Tr}[O_f \rho(t)]$ denotes the decay rate into a final state f , starting from a pure K^0 at $t = 0$: $\rho(t = 0)$ is given by the first matrix in (21), and correspondingly $R(\bar{K}^0 \rightarrow \bar{f}) \equiv \text{Tr}[O_{\bar{f}} \bar{\rho}(t)]$ denotes the decay rate into the conjugate state \bar{f} , starting from a pure \bar{K}^0 at $t = 0$: $\bar{\rho}(t = 0)$ is given by the second matrix in (21). Several relevant asymmetries were defined in [22], including A_T (already introduced above), A_{CPT} , $A_{2\pi}$ and $A_{3\pi}$. We discuss below their possible roles in discriminating between CP- and CPT-violating effects, in particular when CPT violation is invoked so as to mimic CP violation whilst preserving T invariance [26].

In order to parametrize a possible CPT-violating difference in semileptonic-decay amplitudes as postulated there, we define y :

$$\langle \pi^+ e^- \bar{\nu} | \mathcal{T} | \bar{K}^0 \rangle \equiv (1 + y) \langle \pi^- e^+ \nu | \mathcal{T} | K^0 \rangle \quad (23)$$

and we assume that y is *real*, which is justified if the amplitude is T invariant [29]. We assume this here because the purpose of this analysis is to test the hypothesis [26] that the CPLEAR asymmetry can be reproduced by CPT violation alone, retaining T invariance in the mixing: $P_{K\bar{K}} = P_{\bar{K}K}$.³ Another important point [8] is the independence of the asymmetry A_T measured at late times of any possible violation of $\Delta S = \Delta Q$ rule. As seen from [8], violations of this rule may be taken into account simply by introducing the combination

$$\tilde{y} \equiv y + 2\text{Re}(x_-) \quad (24)$$

where, in the notation of [29,8], x_- parametrizes violations of the $\Delta S = \Delta Q$ rule:

$$\langle \pi^+ e^- \bar{\nu} | \mathcal{T} | K^0 \rangle \equiv c + d, \quad \langle \pi^- e^+ \nu | \mathcal{T} | \bar{K}^0 \rangle \equiv c^* - d^* \quad (25)$$

and $x \equiv (c^* - d^*)/(a + b)$, $\bar{x}^* \equiv (c + d)/(a^* - b^*)$, and $x_{\pm} \equiv (x \pm \bar{x})/2$. Again, the hypothesis of T invariance implies the reality of x, \bar{x}, x_{\pm} [29]. If one considers violations of $\Delta S = \Delta Q$ rule, one should take appropriate account of the additional decay modes (25) in A_l (3).

³ For clarity and completeness, we note the following relation between the quantity y defined above and the quantity y defined in [8]: $y = 2\bar{y} \equiv -2b/a$ to lowest order in y and for real a, b [29]: $\langle \pi^+ e^- \bar{\nu} | \mathcal{T} | \bar{K}^0 \rangle = a^* - b^*$, $\langle \pi^- e^+ \nu | \mathcal{T} | K^0 \rangle = a + b$.

In the density-matrix formalism, $y \neq 0$ corresponds to a difference in normalization between the semileptonic observables $O_{\pi l \nu}$ introduced in (20). The analysis of [26], extended in the above straightforward way to take into account of possible violations of the $\Delta S = \Delta Q$ rule, shows that, if one imposes reciprocity, then

$$A_l \simeq -\tilde{y} \quad (26)$$

to lowest order in \tilde{y} .

To make contact with the experimental measurement of the CPLEAR collaboration, one should take into account the different normalizations of the K^0 and \bar{K}^0 fluxes at the production point. Because of this effect, the measured asymmetry [8] becomes:

$$A_T^{exp} = A_l - \tilde{y} \quad (27)$$

The measured [8] value of this asymmetry is:

$$A_T^{exp} \simeq (6.6 \pm 1.3_{stat}) \times 10^{-3} \quad (28)$$

If this experimental result were to be interpreted as expressing CPT violation but T invariance, then \tilde{y} should have the value:

$$\tilde{y} = -(3.3 \pm 0.7) \times 10^{-3} \quad (29)$$

Such a scenario is excluded by the current CPLEAR value of \tilde{y} [19]. The late-time asymmetry measured by CPLEAR can be expressed as [8]:

$$A_T^{exp} \simeq 4\text{Re}(\epsilon) - 2\text{Re}(\tilde{y}) \quad (30)$$

This enables a stringent upper limit to be placed [19]:

$$\frac{1}{2}\tilde{y} = \frac{1}{2}\text{Re}(y) + \text{Re}(x_-) = (0.2 \pm 0.3_{stat}) \times 10^{-3} \quad (31)$$

Therefore, the CPT-violating but T-conserving hypothesis is conclusively excluded independently of any assumption about the validity of the $\Delta S = \Delta Q$ rule.

As a side-remark, we comment on the effect of y on the CPT-violating width difference $\delta\Gamma$, assuming the validity of the $\Delta S = \Delta Q$ rule ($x_- = 0$), which is supported

by [19]. Using $\Gamma_{sl}^L = 0.39 \times \frac{1}{\tau_L} \simeq 8 \times 10^{-18}$ GeV and the value (29) for y , and neglecting any possible other CPT-violating differences in decay rates, we find

$$\delta\Gamma \simeq 1.06 \times 10^{-19} \text{ GeV} \quad (32)$$

which makes the following contribution to $Re(\delta)$:

$$2\text{Re}(\delta) = \frac{\delta\Gamma|\Delta\Gamma|}{|\Delta\Gamma|^2 + 4|\Delta m|^2} = \frac{\delta\Gamma \cos^2\phi}{|\Delta\Gamma|} \simeq 6.8 \times 10^{-6} \quad (33)$$

where we have used $\phi \simeq 43.49^\circ \bmod \pi$. This contribution is far below the present experimental sensitivity discussed below.

Next, we comment on the possibility that what we usually regard as CP violation in the mass matrix is actually due to CPT violation. In such a case, one would have to set $|\epsilon| \rightarrow 0$ and make the following choices for the CPT-violating mixing parameters

$$\text{mimic CP violation :} \quad \delta M = 0, \quad \widehat{\delta\Gamma} \rightarrow \frac{2|\epsilon|}{\cos\phi}, \quad (34)$$

On account of (18), then, the observable A_T would have the following time-independent first-order expression:

$$A_T = 2|X'| \cos(\phi - \phi_{X'}) + 2|X| \cos(\phi + \phi_X) = 4|\epsilon| \cos\phi, \quad (35)$$

which is identical to the conventional case of CPT symmetry. However, this is not the case for all observables, for instance the A_{CPT} asymmetry, defined by setting $f = \pi^- e^+ \nu$, $\bar{f} = \pi^+ e^- \bar{\nu}$ in (22). In particular, one has the following asymptotic formula for A_{CPT} :

$$A_{\text{CPT}} \rightarrow 4 \sin\phi \cos\phi \widehat{\delta M} - 2 \cos^2\phi \widehat{\delta\Gamma}. \quad (36)$$

which would yield the following asymptotic prediction under the “mimic” assumption (34):

$$A_{\text{CPT}} \rightarrow -4|\epsilon| \cos\phi, \quad (37)$$

to be contrasted with the standard result that $A_{\text{CPT}} = 0$ in the absence of CPT violation.

For comparison with experimental data of CPLEAR, it is useful to express the conventional CPT-violating parameter δ (10) in terms of $\widehat{\delta\Gamma}$:

$$\text{Re}(\delta) = \frac{1}{2}\widehat{\delta\Gamma}\cos^2\phi = |\epsilon|\cos\phi > 0, \quad \text{Im}(\delta) = -\frac{1}{2}\widehat{\delta\Gamma}\sin\phi\cos\phi \quad (38)$$

The experimental asymmetry A_T^{exp} (27), then, would be obtained upon the identification of A_T in (35) with A_l ,

$$A_T^{exp} = 4\text{Re}(\delta) - \tilde{y} \quad (39)$$

Notice that in principle such a situation is consistent with the experimental data, given that the combination $A_T^{exp} + \tilde{y} = 4\text{Re}(\delta) > 0$. Taking into account (28), (31) and (39) we observe that the mimic requirement would imply

$$\text{Re}(\delta)_{mimic} \sim (1.75 \pm 0.7) \times 10^{-3} \quad (40)$$

However, the CPLEAR Collaboration has measured [18] $\text{Re}(\delta)$ using the asymptotic value of the asymmetry A_δ :

$$A_\delta \equiv \frac{\overline{R}_+ - R_-(1 + 4\text{Re}\epsilon_L)}{\overline{R}_+ + R_-(1 + 4\text{Re}\epsilon_L)} + \frac{\overline{R}_- - R_+(1 + 4\text{Re}\epsilon_L)}{\overline{R}_- + R_+(1 + 4\text{Re}\epsilon_L)} \quad (41)$$

which asymptotes at large times to $-8\text{Re}(\delta)$, independently of any assumption on the $\Delta S = \Delta Q$ rule:

$$\text{Re}(\delta) \simeq (3.0 \pm 3.3_{stat} \pm 0.6_{syst}) \times 10^{-4} \quad (42)$$

in apparent conflict with (40).

The fact that the CP violation seen in the mass matrix cannot be mimicked by CPT violation [4] has been known for a long time. The possible magnitude of CPT violation is constrained in particular by the consistency between ϕ_{+-} and the superweak phase ϕ_{sw} . However, it is possible to mimic CP violation in any particular observable by a suitable choice of δ . For example, as was shown in [22], the standard superweak result for $A_{2\pi}$ may be reproduced by setting $|\epsilon| \rightarrow 0$ and using (34), which give $|X| \rightarrow |\epsilon|$ and $\phi_X = 0$. The standard CP-violating result for $A_{3\pi}$ may also be obtained with the choices (34) [22], which give $|X'| \rightarrow |\epsilon|$ and $\phi_{X'} = \pi$, since $\tan(\phi - \pi) = \tan\phi$. But, as already emphasized, the dynamical equations determining the density matrix prevent all observables from being mimicked in this way: this is what we found above with the A_{CPT} observable (37), to be contrasted with the

standard result $A_{\text{CPT}} = 0$. Moreover, as mentioned above, the mimic hypothesis is excluded by the recent CPLEAR result (42).

It was also pointed out previously [22,30] that deviations from conventional closed-system quantum mechanics of the type discussed in [20], which lead to stochastic CPT violation, also cannot account for the CP violation observed in the neutral kaon system. We remind the reader that generic possible deviations from closed-system quantum-mechanical evolution in the neutral kaon system - which might arise from quantum gravity or other stochastic forces - may be described by the three real parameters α, β, γ of [20], if one assumes energy conservation and dominance by $\Delta S = 0$ stochastic effects. These parameters lead to entropy growth, corresponding to the appearance of an arrow of time and violation of CPT [31], as has sometimes been suggested in the context of a quantum theory of gravity. However, this CPT violation cannot be cast in the conventional quantum-mechanical form discussed above. The most stringent bounds on the stochastic CPT-violating parameters α, β, γ have been placed by the CPLEAR collaboration [32]. They are not far from the characteristic magnitude $\mathcal{O}(M_K^2/M_P)$, where the Planck mass $M_P \simeq 10^{19}$ GeV, near the scale at which such effects might first set in [21] if they are due to quantum-gravitational effects.

5 The KTeV Asymmetry in $K_L \rightarrow e^- e^+ \pi^- \pi^+$ and its Interpretation

Subsequent to the CPLEAR analysis, the KTeV Collaboration has reported [9] a novel measurement of a T-odd asymmetry in the decay of $K_L \rightarrow e^- e^+ \pi^- \pi^+$. Since incoming and outgoing states are *not* exchanged in the KTeV experiment, unlike the CPLEAR measurement comparing $\bar{K}^0 \rightarrow K^0$ and $K^0 \rightarrow \bar{K}^0$ transitions, it cannot provide direct evidence for T violation. However, it is interesting to discuss the information this measurement may provide about CP, T and CPT symmetry.

This decay has previously been analyzed theoretically in [33], assuming CPT symmetry. The decay amplitude was decomposed as:

$$\mathcal{M}(K_L \rightarrow \pi^+ \pi^- e^+ e^-) = \mathcal{M}_{Br} + \mathcal{M}_{M1} + \mathcal{M}_{E1} + \mathcal{M}_{SD}^{V,A} + \mathcal{M}_{CR} \quad (43)$$

and the various parts of the amplitude (43) have the following interpretations:

- \mathcal{M}_{Br} : Amplitude for the Bremsstrahlung process related to the standard CP-violating $K_L \rightarrow 2\pi$ amplitude, violating CP just like the conventional ϵ parameter. This amplitude is proportional to a coupling constant [33]

$$g_{Br} = \eta_{+-} e^{i\delta_0(M_K^2)} \quad (44)$$

where η_{+-} is the conventional CP-violating parameter, whose phase ϕ_{+-} is that of $K_L \rightarrow \pi^+\pi^-$: $\delta_0(M_K^2)$ is the relevant $I = 0$ $\pi^+\pi^-$ phase shift.

- \mathcal{M}_{M1} : The magnetic-dipole contribution to the amplitude, which is CP-conserving. The corresponding coupling constant has a non-trivial phase [33]:

$$g_{M1} = i|g_{M1}|e^{i\delta_1(m_{\pi\pi})+\delta\varphi} \quad (45)$$

where δ_1 is the $\pi\pi$ P -wave phase shift. The amplitude is invariant under CPT if $\delta\varphi = 0$, leaving the prefactor i as a consequence of CPT invariance. The estimate $|g_{M1}| = 0.76$ is given in [33].

- \mathcal{M}_{E1} : This denotes the electric-dipole contribution. It is CP-conserving, and its coupling constant g_{E1} has been computed in [33]. Its phase is related to that of g_{M1} via $\arg(g_{E1}/g_{M1}) \simeq \phi_{+-}$.

- $\mathcal{M}_{SD}^{V,A}$: This is the contribution originating in the short-distance Hamiltonian describing the transition $s\bar{d} \rightarrow e^+e^-$. Its coupling constant has been calculated in the Standard Model [33], with the result:

$$g_{SD} = i(5 \times 10^{-4})\sqrt{2}\frac{M_K}{f_\pi}e^{i\delta_1(m_{\pi\pi})} \quad (46)$$

where f_π is the pion decay constant. One could in principle introduce CPT violation into this amplitude by allowing $A(\bar{K}^0 \rightarrow \pi^+\pi^-e^+e^-) \neq A(K^0 \rightarrow \pi^+\pi^-e^+e^-)$. As seen in (46), these amplitudes may be related to $M_{\bar{K}}$ and M_K respectively, which could be different if CPT is violated.

- \mathcal{M}_{CR} : This denotes the CP-conserving contribution due to a finite charge radius of the K^0 . Its coupling g_P has the phase of $K_S \rightarrow \pi^+\pi^-$.

The KTeV [9] Collaboration's measurement is of a CP-violating asymmetry \mathcal{A} in the angle Φ between the vectors normal to the e^-e^+ and $\pi^+\pi^-$ planes [33], which is related to the particle momenta by:

$$\sin\Phi\cos\Phi = \eta_l \times \eta_\pi \cdot \left(\frac{p_+ + p_-}{|p_+ + p_-|} \right) \cdot (\eta_l \cdot \eta_\pi) \quad (47)$$

where the unit vectors $\eta_{l,\pi}$ are defined as $\eta_l \equiv k_+ \times k_- / |k_+ \times k_-|$ and $\eta_\pi \equiv \frac{p_+ \times p_-}{|p_+ \times p_-|}$, with k_\pm the lepton momenta and p_\pm the pion momenta. The observable is a CP asymmetry \mathcal{A} of the process, which we shall discuss below. The Φ distribution $d\Gamma/d\Phi$

may be written in the following generic form [33]:

$$\frac{d\Gamma}{d\Phi} = \Gamma_1 \cos^2 \Phi + \Gamma_2 \sin^2 \Phi + \Gamma_3 \cos \Phi \sin \Phi \quad (48)$$

where the last term changes sign under the CP transformation and is T-odd, i.e., it changes sign when the particle momenta are reversed. However, it clearly does not involve switching ‘in’ and ‘out’ states, and so is not a direct probe of T violation ⁴.

A detailed functional form for Γ_3 is given in [33]. Following the above discussion of the various terms in the decay amplitude (43), this term is interpreted [33] in terms of the dominant Bremsstrahlung, magnetic-dipole and electric-dipole contributions. For our purposes, it is sufficient to note that it involves the coupling constant combinations $\text{Re}(g_{M1}g_{BR}^*)$ and $\text{Re}(g_{M1}g_{E1}^*)$, which involve amplitudes with different CP properties, and hence violate CP manifestly. It depends in particular on the phase ϕ_{+-} of the conventional CP-violating $K_L \rightarrow \pi^+\pi^-$ decay amplitude, via the K_1 admixture in the K_L wave function, which enters in the M1 amplitude for $K_L \rightarrow \pi^+\pi^-\gamma$. The following is the generic structure of the integrated asymmetry measured by KTeV [33]:

$$\mathcal{A} = \frac{\int_0^{\pi/2} \frac{d\Gamma}{d\Phi} d\Phi - \int_{\pi/2}^{\pi} \frac{d\Gamma}{d\Phi} d\Phi}{\int_0^{\pi/2} \frac{d\Gamma}{d\Phi} d\Phi + \int_{\pi/2}^{\pi} \frac{d\Gamma}{d\Phi} d\Phi} \simeq \mathcal{A}_1 \cos \Theta_1 + \mathcal{A}_2 \cos \Theta_2 \left| \frac{g_{E1}}{g_{M1}} \right| \quad (49)$$

where

$$\Theta_1 \equiv \phi_{+-} + \delta_0 - \bar{\delta}_1 - \frac{\pi}{2} - \overline{\delta\varphi} \bmod \pi, \quad \Theta_2 \equiv \phi_{+-} - \frac{\pi}{2} - \overline{\delta\varphi} \bmod \pi \quad (50)$$

and $\bar{\delta}_1$ and $\overline{\delta\varphi}$ are averages of the $\pi\pi$ P -wave phase shift and $\delta\varphi$, respectively, in the region $m_{2\pi} < m_K$. Numerical estimates of the quantities $\mathcal{A}_{1,2}$ in terms of the different couplings in (43) were given in [33]:

$$\mathcal{A}_1 \simeq 0.15, \quad \mathcal{A}_2 \simeq 0.38, \quad (51)$$

⁴ It is generally agreed that final-state electromagnetic interactions can be neglected for present purposes. The KTeV collaboration has recently reported [34] a null asymmetry in the angle between the $\pi^+\pi^-$ and e^+e^- planes in the Dalitz decay $K_L \rightarrow \pi^+\pi^-(\pi^0 \rightarrow e^+e^-\gamma)$. This provides a nice check on the experimental technique, but does not test directly the structure of the final-state interactions, since the π^0 decays outside the Coulomb fields of the $\pi^+\pi^-$ pair.

leading to the following prediction for \mathcal{A} :

$$\mathcal{A} \simeq 0.15 \sin [\phi_{+-} + \delta_0(m_K^2) - \bar{\delta}_1] \quad (52)$$

if the CPT-violating phase $\bar{\delta}\varphi = 0$. Using the experimental values $\delta_0 \simeq 40^\circ$, $\bar{\delta}_1 \simeq 10^\circ$ and $\phi_{+-} \simeq 43^\circ$, (52) becomes

$$\mathcal{A} \simeq 0.14 \quad (53)$$

As already mentioned, the experimental value

$$\mathcal{A}_{exp} = (13.5 \pm (2.5)_{stat} \pm (3.0)_{syst}) \% \quad (54)$$

agrees very well with the theoretical prediction (53) obtained assuming the CPT-violating phase $\bar{\delta}\varphi = 0$.

We now analyze how well this measurement tests CPT, and assess how this test compares with other tests. Consider first the Bremsstrahlung contribution: as mentioned above, the coupling g_{br} has a phase ϕ_{+-} . In principle, CPT violation in the neutral-kaon mass matrix could shift this phase away from its superweak value ϕ by an amount $\delta\phi$:

$$|m_{K^0} - m_{\bar{K}^0}| \simeq 2\Delta m \frac{|\eta_{+-}|}{\sin\phi} |\delta\phi| \quad (55)$$

where (as always) we neglect effects that are $\mathcal{O}(\epsilon')$, and we recall that $|\eta_{+-}| \simeq |\epsilon|/\cos\delta\phi \simeq |\epsilon|$. The best limit on such a mass difference is now provided by the CPLEAR experiment [5]:

$$|m_{K^0} - m_{\bar{K}^0}| \leq 3.5 \times 10^{-19} \text{ GeV (95\% C.L.)}, \quad (56)$$

The limit (56) determines $|\delta\phi| \lesssim 0.86^\circ$, whereas a combination of previous data from the NA31, E731 and E773 Collaboration yields [22] $\delta\phi \lesssim (-0.75 \pm 0.79)^\circ$. Such a phase change $|\delta\phi|$ would change \mathcal{A} by an amount $|\delta\mathcal{A}| \lesssim 10^{-3}$, far smaller than the experimental error in (54), and also much smaller than the likely theoretical uncertainties.

We consider next the magnetic-dipole contribution, with the possible incorporation of a CPT-violating phase $\bar{\delta}\varphi$ (45). To first order in $\bar{\delta}\varphi$, the corresponding

change in \mathcal{A} is:

$$\delta\mathcal{A} \simeq \left(0.15\sin\Theta_1^{(0)} + 0.38\sin\Theta_2^{(0)} \left| \frac{g_{E1}}{g_{M1}} \right| \right) \overline{\delta\varphi} \quad (57)$$

with $\Theta_{1,2}^{(0)}$ evaluated using (50) and assuming $\overline{\delta\varphi} = 0$. However, this small-angle approximation is not justified, so we use the full expression (49) for \mathcal{A} , and interpret the experimental value (54) as implying that $\mathcal{A} \gtrsim 0.096$ at the one-standard-deviation level, corresponding to

$$0.14\cos\overline{\delta\varphi} - 0.04\sin\overline{\delta\varphi} \geq 0.096 \quad (58)$$

which leads to

$$-70^\circ \lesssim \overline{\delta\varphi} \lesssim +40^\circ \quad (59)$$

for the allowed range of this CPT-violating parameter, where we have used the estimate [33] $|g_{E1}/g_{M1}| \simeq 0.05$, and not made any allowance for theoretical uncertainties.

The range (59) is clearly much wider than the corresponding scope for a CPT-violating contribution $\delta\phi$ to the phase ϕ_{+-} of η_{+-} , and the range would be larger still if we expanded the allowed range of $\overline{\delta\varphi}$ to the 95% C.L. limits. We also note in passing that the magnitude of the short-distance contribution (46) is so small that no interesting limit on direct CPT violation in it can be obtained.

We now address the question whether all the KTeV asymmetry could be due to CPT violation. This would occur if

$$\mathcal{A}_1 \cos\Theta_1^{(0)} + \mathcal{A}_2 \cos\Theta_2^{(0)} \left| \frac{g_{E1}}{g_{M1}} \right| = 0 \quad (60)$$

This possibility is disfavoured by the theoretical estimates of $\mathcal{A}_{1,2}$, but cannot be logically excluded. If (60) were to hold, the KTeV asymmetry could be written in the form

$$\mathcal{A} \simeq \mathcal{A}_1 \sin\overline{\delta\varphi} \left[\sin\Theta_1^{(0)} - \cos\Theta_1^{(0)} \tan\Theta_2^{(0)} \right] \quad (61)$$

in which case the experimental value (54), at the one-standard-deviation level, would be reproduced if

$$0.13 \lesssim \mathcal{A}_1 \sin\overline{\delta\varphi} \lesssim 0.22 \quad (62)$$

Unfortunately, the amplitude \mathcal{A}_1 has not yet been measured experimentally. However, if one adopts the estimate that $\mathcal{A}_1 = 0.15$ as in (51), then the KTeV asymmetry could be reproduced if $\overline{\delta\varphi} \gtrsim 58^\circ$.

We conclude that, whilst *a priori* it may seem very unlikely that the KTeV asymmetry could be due to CPT violation, we are unable to exclude rigorously this possibility at the present time. We hope that future measurements of this and related decay modes will soon be able to settle this issue.

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